Proseminar Advanced Complexe Analysis

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Extra problems

1. Let $\gamma : [0,1] \to \mathbb{C}$ be a curve starting at $\gamma(0) = a$ and $f_a \in \mathcal{O}_a$ a germ. If f_a has an analytic continuation along $\gamma|_{[0,t]}$ we define $r(t)$ to be the radius of convergence of the power series of $f_{\gamma(t)}$ with center at $\gamma(t)$. Otherwise, if there is no analytic continuation, we set $r(t) = 0$.

   Show that if $r(t) = \infty$ for some $t \in [0,1]$, then $r(t) = \infty$ for all $t \in [0,1]$. Otherwise show that $r : [0,1] \to [0,\infty)$ is continuous.

2. Let X be a pathwise connected topological space and $x_0, x_1 \in X$. Show that all paths from x_0 to x_1 are homotopic iff every loop is null-homotopic.

3. Let $U \subseteq \mathbb{C}$ be a simply connected domain. If $f \in \mathcal{H}(U)$ is nowhere-vanishing in U, then by [R, Thm. 4.8] there exist $g \in \mathcal{H}(U)$ such that $\mathrm{e}^g = f$. Characterize the set of all g with this property.

4. Consider the curve $\gamma(t) = \mathrm{e}^{2\pi\mathrm{i}t}$, $t \in [0,1]$. Find domains U_1, U_2 such that $\gamma \sim_{U_1} 0$ as well as $\gamma \not\sim_{U_2} 0$. Find a curve γ and a domain U such that $\gamma \sim_U 0$ but γ is not null-homotopic in U.

References

[J] K. Jänich, *Funktionentheorie*, Springer, 2004

[R] A. Rainer, *Advanced Complex Analysis*, Lecture notes, 2017.